Trigonometry practice in a book published in the year 1930...

20. Discuss the solution of $\cos^2 x - 2m\cos x + 4m^2 + 2m - 1 = 0$ for various values of m.

21. Show that if $bh\cos\theta + ak\sin\theta = ab$ has roots for $\cos\theta$, they always determine values of θ.

22. Express $\sin\frac{\theta}{2}$ in terms of $\sin\theta$, when θ is in the neighbourhood of $420°$. For what precise neighbourhood is the result valid ?

23. Prove that $\tan\frac{\theta}{4}$ is one of the values of $\frac{1\pm\sqrt{(1-\sin\theta)}}{1\pm\sqrt{(1+\sin\theta)}}$, and find the other values.

24. Prove $\cos\frac{\theta}{2} = (-1)^{\left[\frac{\theta+\pi}{2\pi}\right]}\sqrt{\{\frac{1}{2}(1+\cos\theta)\}}$. (See footnote, p. 46.)

25. If p is an integer and $-1 < q < 1$, find the number of possible values of $\sin x$, such that (i) $\sin 2px = q$, (ii) $\sin(2p+1)x = q$.

26. Solve $x^5 - 5k^2x^3 + 5k^4x = 2k^5\cos a$, for x in terms of a and k.

27. Simplify $\tan^{-1}\frac{p-q}{1+pq} + \tan^{-1}\frac{q-r}{1+qr}$.

28. Prove that $\tan^{-1}\frac{1}{p} = \tan^{-1}\frac{1}{p+q} + \tan^{-1}\frac{q}{p^2+pq+1}$.

29. Use the result of No. 28 to express $\frac{\pi}{4}$ in the form $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$.

Also express $\tan^{-1}\frac{1}{2}$ and $\tan^{-1}\frac{1}{3}$ each in the form $\tan^{-1}\frac{1}{m} + \tan^{-1}\frac{1}{n}$ where m and n are positive integers.

30. Prove that $\frac{\pi}{4} = 2\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{1}{13}$.

31. Prove that $\frac{\pi}{4} = 2\cot^{-1}5 + \cot^{-1}7 + 2\cot^{-1}8$.

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