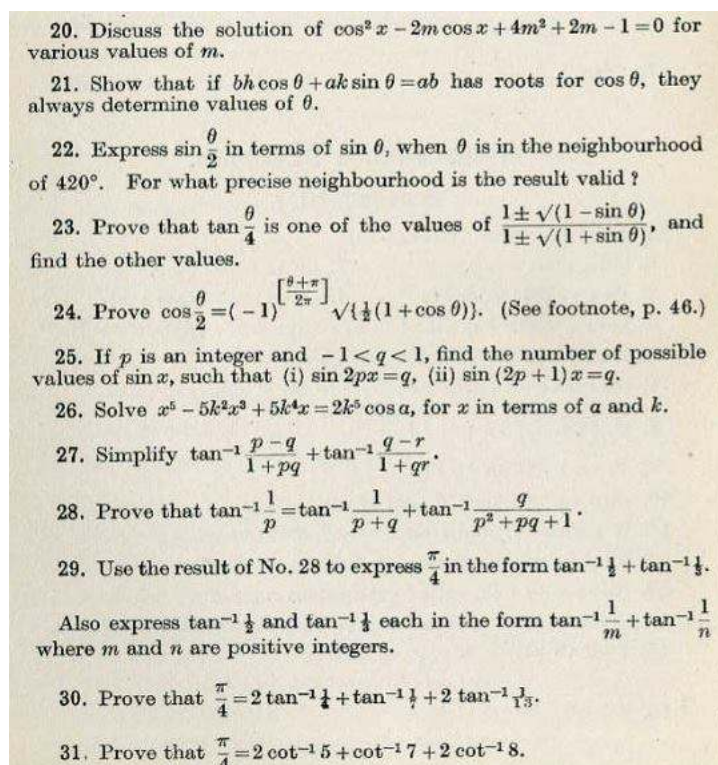


Trigonometry practice in a book published in the year 1930...



20. Discuss the solution of $\cos^2 x - 2m \cos x + 4m^2 + 2m - 1 = 0$ for various values of m .

21. Show that if $bh \cos \theta + ak \sin \theta = ab$ has roots for $\cos \theta$. They always determine values of θ .

22. Express $\sin \frac{\theta}{2}$ in terms of $\sin \theta$, where θ is in the neighbourhood of 420° .

For what precise neighbourhood is the result valid ?

23. Prove that $\tan \frac{\theta}{4}$ is one of the values of $\frac{1 \pm \sqrt{1 - \sin \theta}}{1 \pm \sqrt{1 + \sin \theta}}$, and find the other values.

24. Prove that $\cos \frac{\theta}{2} = (-1)^{\left[\frac{\theta+\pi}{2\pi}\right]} \sqrt{\frac{1}{2}(1 + \cos \theta)}$, $[x]$ =greatest integer smaller or equal to x

25. If p is an integer and $-1 < q < 1$, find the number of possible values of $\sin x$, such that
(i) $\sin 2px = q$, (ii) $\sin(2p+1)x = q$.

26. Solve $x^5 - 5k^2x^3 + 5k^4x = 2k^5 \cos \alpha$, for x in terms of α and k .

27. Simplify $\tan^{-1} \frac{p-q}{1+pq} + \tan^{-1} \frac{q-r}{1+qr}$.

28. Prove that $\tan^{-1} \frac{1}{p} = \tan^{-1} \frac{1}{p+q} + \tan^{-1} \frac{q}{p^2+pq+1}$.

29. Use the result of No.28 to express $\frac{\pi}{4}$ in the form of $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3}$.

Also express $\tan^{-1} \frac{1}{2}$ and $\tan^{-1} \frac{1}{3}$ each in the form of $\tan^{-1} \frac{1}{m} + \tan^{-1} \frac{1}{n}$ where m and n are positive integers.

30. Prove that $\frac{\pi}{4} = 2\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{7} + 2\tan^{-1} \frac{1}{13}$.

31. Prove that $\frac{\pi}{4} = 2\cot^{-1}5 + \cot^{-1}7 + 2\cot^{-1}8$.